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ABSTRACT

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VISUALIZATION AND ARITHMETIC PROBLEM SOLVING

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Abstract

One difficulty that mathematically naive subjects encounter in solving arithmetic word problems involves the limitation on short term memory capacity. It is hypothesized that naive subjects, not having access to formal problem solving strategies, may find visualization useful in reducing the strain on STM. Two experiments are reported. The results of the first support the hypothesis that naive subjects make fewer errors on problems that are easier to visualize (problems using small numbers and discrete quanfities) than on problems which are harder to visualize (those using large numbers and continuous quantities) when all other factors are held constant. The second experiment replicates the results of the first and also supports the hypothesis that this effect is not found with experienced problem solvers. The educational implications of these findings are briefly discussed.

Introduction

The research reported in this paper developed out of a concern about the difficulties encountered by the average adult when confronted with simple arithmetic word problems. Very little research has been done on the techniques of problem solving employed by non-academic adults. Of course many of the conclusions from studies using other populations of subjects are equally valid for this one, but there are also important differences. Adults, unlike children, have fully developed language competence and extensive real life experiences. On the other hand, they often lack the kind of formal problem solving strategies available to undergraduate or graduate students at elite universities. If these adult students are to be helped effectively, there must be a basic understanding of their skills and deficits, of the processes they use and the difficulties they encounter.

One of the most important difficulties that adult students often mention in discussions of arithmetic word problem solving is the difficulty of keeping necessary information in mind while solving the problem. This difficulty has been apparent both in my classroom experience and in an earlier exploratory study I did in which subjects solved arithmetic word problems in an interview setting. It is a difficulty which has received relatively little attention in the literature.

That information retention is a problem should hardly come as a surprise. It is well known that short term memory (STM) has a limited capacity — 7 ± 2 chunks if George Miller's (1956) magic number is correct — even when all that is required is retention. In order to solve problems, it is not only necessary to keep information in STM, but to process it simultaneously. Short term memory is known to be highly unstable, being subject to both spontaneous decay and loss by interference. In the solving of arithmetic word problems, both of these factors play a role. Unless the problem in question can be solved with extreme rapidity, the information will become unavailable before it can be used. The act of rehearsal can help to keep it in mind, but will make it difficult to process simultaneously. Processing interferes with rehearsal, so that the solver loses his grasp of the essential facts.

Newell and Simon (1972) recognized the problem. They point out in their discussion of the role of STM in problem solving that:

> ... only a very few symbols can be retained in STM — perhaps not more than two — when even a simple processing task (e.g., counting backwards by threes) is interposed between presentation and recall. And almost all tasks that require attention, in some intuitive sense, subtract from the effective capacity of STM as measured by standard tests.

But then they dismiss this factor as an insignificant detail:

> Since the problem solving activity is self-paced, the problem solver can adjust his rate and style of processing information so that he does not appear either to have a rapidly decaying STM or to rehearse.

It is at this point that the choice of subjects makes an important difference. There is absolutely no reason to suppose that adult subject populations differ in short term memory capacity. They obviously do differ, however, in the strategies available to them for solving these problems. While Newell and Simon may well be right that the problem solvers do not experience cognitive strain from an overloaded STM, this is because the strategies their subjects have available allow them to chunk the information in such a way that it can be maintained during processing. The mathematically naive subject has no such options.

Most investigators agree that problem solving can be broken down into several steps or stages (Polya, 1945; Simon & Hayes, 1976; Greeno, 1973). For the purpose of this study, I shall refer to the stages as comprehension, planning, computation and checking, and shall mainly be concerned with the first two. The computation stage is of interest here only insofar as it interfers with planning, and checking will not be considered at all.

A typical example of a subject unable to carry out a problem solution because of STM overload can be seen in the following excerpt from one of the protocols of an earlier informal investigation:

> At the beginning I knew what I was going to do. I knew what I had

> to find. I think that what probably I did, I let the sixty cents and the twelve hundred miles and the forty just run around in my head and not know where to put them.

Yet subjects like this one do solve some problems correctly. Although they may have difficulty because of their lack of formal strategies, it is clear that some informal mechanisms are at work.

It is the hypothesis of this study that visualization may be one of these informal strategies. If this is the case, then arithmetic word problems that are more easily visualizable will be more readily solved by these subjects than those more difficult to visualize, all other factors being equal. The following two experiments were designed to test the hypothesis that visualizability plays an important role in problem solving among mathematically naive solvers.

Experiment I

In order to test the hypothesis that easily visualizable arithmetic word problems would be easier for mathematically naive subjects to solve than formally identical problems that are more difficult to visualize, it was necessary to devise problems that would differ in visualizability but not in other ways that might increase difficulty.

Although there is no generally accepted characterization of what is visualizable and what is not, there are a number of fairly obvious properties that could be used. For example, a problem dealing with real objects or events would be easier to visualize than one using nonsense words or names; a small number of objects should be easier to visualize than a large number. It also seems reasonable to assume that objects are easier to picture than events. Discrete quantities should be easier to visualize than continuous ones (e.g. three apples vs. three pounds of apples.)

In this experiment, three notions of visualizability were used, with close attention to extraneous difficulties thus introduced. A questionnaire accompanied the test so that subjective awareness could be compared with objective results. Nevertheless, it is clear that the results of the experiment cannot be taken as evidence that visualization as such is part of the problem solving process — only that problems which are easier to visualize are easier to solve.

Method

Subjects. The 36 subjects for this experiment were recruited from a college serving an inner city, adult student body. These subjects saw mathematics as a major obstacle in their academic career and many participated in the study because of their interest in the difficulties of math learning. None of the subjects had taken mathematics courses beyond elementary algebra, and many did not even have algebra. All subjects were paid at the standard rate.

Materials. The structure of all problems used can be expressed by one of the following equations:

$\frac{a\,b}{c} = d$ or $\frac{c}{a\,b} = d$ where a, b and c are given and d is to be found.

That is, each problem involved two steps: one multiplication and one division.

Each of the problems used in Experiment I and II was produced in four different version, using large and small numbers and discrete and continuous quantities: (a) small numbers and discrete quantities, (b) small numbers and continuous quantities, (c) large numbers and discrete quantities and (d) large numbers and continuous quantities. It was expected that small numbers and discrete quantities would each be easier to visualize. The large numbers used were ten times the samll numbers. (e.g., 3 and 30 for small and large respectively.) This strategy was expected to minimize the additional computational difficulty of working with larger numbers.

Since discrete quantities are those which can be counted (e.g., 2 dogs, 7 jars, etc.) while continuous quantities are those which are measured (e.g., 5 gallons, 6 feet, etc.), it seems intuitively clear that discrete are easier to visualize. Five pounds of apples can be visualized as a bag of apples, but will not capture the "fiveness" in the same way that five apples would. Of course this is not necessarily true in all cases, some continuous quantities having taken on discrete connotations from experience (e.g., five quarts of milk are readily visualizable as five cartons). Care was taken to use continuous quantities which could not be readily imaged as discrete.

Continuous quantities are also more difficult than discrete because there is an extra element to remember and process — the unit of measurement. This difficulty was dealt with by using discrete quantities that involved as many elements of information as the continuous (e.g., five packs of cigarettes vs. five ounces of tobacco.) Another difficulty with continuous quantities is the occasional need to convert units (e.g., inches to feet, ounces to pounds, etc.). No such conversions were called for in the problems used here.

Five problem stories were generated for Experiment I, each being produced in the four conditions mentioned above. The story was kept as constant as possible across the four conditions, with only minor changes made for the sake of plausibility. An example of the small number-discrete quantity version and the large number-continuous quantity version of the first problem is given below:

1a. A man finds that he has smoked 9 packs of cigarettes. For 3 packs of cigarettes he pays $2. How much has he paid for the cigarettes he has smoked?

1d. A man finds that he has smoked 90 ounces of pipe tobacco. For 30 ounces of tobacco he pays $20. How much has he paid for the tobacco he has smoked?

To explore the effect of using nonsense words instead of familiar nouns, the last problem was the following:

5a. A machine produces 2 pieces of frell from 8 bars of grint. Each bar of grint costs $3. How much does it cost to produce each piece of frell?

Altogether, then, there were five problems, each of which had four versions. A given subject saw only one version of each problem. A latin square design was used so that each subject saw each of the five different problems and each of the four different conditions; one condition appearing twice, once in the nonsense problem and again in one of the other four.

Each subject was given a packet consisting of an instruction sheet, a practice problem, five arithmetic word problems, and a questionnaire. The questionnaire asked subjects whether they formed mental images and if so to describe them.

Procedure. Testing was conducted in groups ranging in size from five to twenty subjects. A large digital clock showing minutes and seconds was placed in clear view of the subjects. The instructions were read aloud as the subjects followed on their written copies. Then the subjects were asked to work the practice problem, writing down the time on the clock before beginning reading and again immediately upon recording the answer. The experimenter monitored this process to make sure all subjects were able to see the clock, read it correctly and record the time quickly and accurately. Subjects were also instructed to answer the questionnaire as carefully as possible after completing the problems.

Results. The overall error rate for the four main problems was 35%. The problem involving nonsense words was analyzed separately since the error rate (75%) was too high to warrent inclusion in the general analysis.

Table 1 shows the mean error rates for the four conditions in Problems #1 - #4.

Table 1: Error Rates

	Small Numbers	Large Numbers	
Discrete Quantities	31%	31%	31%
Continuous Quantities	25%	53%	39%
	28%	42%	35%

The main hypothesis of the experiment concerned the effects of visualizability on ease of problem solving. Both large numbers and continuous quantities were expected to make visualization more difficult than small numbers or discrete quantities. However, there was no main effect for either the small-large or the continuous-discrete factor. The interaction, however, was significant: $\chi^2_{(1)} = 4.08$, $p < .05$. The four problems also differed significantly in difficulty: $\chi^2_{(3)} = 16.24$, $p < .005$. There was no interaction between the four problems and the four conditions; that is, the effect of problem difficulty did not interact with the effect of condition difficulty.

Solution Times. An analysis of variance was carried out on the solution times of the correctly solved problems. One main effect was found: problems with continuous quantities took significantly longer than those with discrete quantities: $F(1, 8) = 10.49$, $MS = 2934$, $p < .025$. Although the interaction between large-small and continuous-discrete was not significant, the pattern of solution times was similar to that for errors.

Questionnaire. The questionnaire provided some interesting information about subjects' conscious experiences of visualization. Of the 36 subjects, 21 claimed to experience mental images while working,

There was no correlation between the four conditions and the reported ease of visualization. Subjects did report, however, that those problems which were easier to solve, as measured by the error rate per problem, were easier to visualize.

Subjects' descriptions of mental images were often too general to be very useful. Responses tended to be very brief: "berries," or "cigarettes " or "balls of yarn," for example. A few, however, gave some indication of how images might actually be used in problem solving. For example, one subject said, "could visualize 2 balls of yarn made into a pair of socks" for the problem in which 4 pairs of socks were made from 8 balls of yarn. Another said, "I pictured the nine packs of cigarettes in groups of 3's." for the problem in which a man smokes 9 packs of cigarettes and pays $2 for every 3 packs. Others showed a different use of visualization which took the form of seeing the problem situation rather than the quantities that the problem specifically dealt with. An example of such an image is "a man at the counter buying cigarettes," or "a tie rack stating 1/2 off price of $24.00 each."

Although none of the subjects claimed that Problem #5 was easy to visualize, three subjects reported making the attempt, one unsuccessfully: "I tried to use wood as an example but became lost in the process." The other two were more successful: "large machines with rolls — producing the frell" and the more elaborate, "frell — seemed like ribbon candy and grint (the liquid it came from) seemed like it was in a big vat and the frell poured from it." It is interesting to note, however, that none of these three subjects solved the problem correctly.

Discussion

The main hypothesis of this experiment was that visualizability would make problems easier to solve. The two factors influencing visualizability were size of numbers and descrete vs. continuous quantities. It was expected that either of these factors might independently make the problems more difficult aside from any effect they might have on visualizability. As stated above, every effort was made to minimize these difficulties. The measures taken to do so seem to have been successful since neither continuous quantities nor large numbers by themselves caused an increase in errors. Problems containing continuous quantities did take longer to solve than those involving discrete, but this is more likely due to the subjects' lack of confidence in dealing with measurements and thus increasing reading and checking time. The fact that there were not more errors indicates that whatever caused the delay did not in fact hinder solution strategies.

The results showed, however, that although neither large numbers nor continuous quantities were individually responsible for increasing the error rate, the two in combination did cause significantly more difficulty, implying that a single mechanism may be involved. Whether or not this mechanism is actual visualization, it will be assumed that it is responsive to how visualizable a problem is. It is not difficult to imagine how actual visualization might be affected by the combination of two difficulties but not by either one individually. In the case of the relatively simple problems used here, it might be possible to visualize a small number of indistinct representations, or a large indistinct number of discrete objects, but if both the number and the object lacked clear definition the task would become unmanagable.

It is important to note that not all factors which increased the difficulty of problems interacted in this way. As mentioned before, the problems varied significantly in difficulty, but there was no interaction between problem difficulty and the visualizability conditions. Since the problems were designed to represent equally visualizable situations and varied in difficulty for other reasons (see Horwitz, 1980), this lack of interaction tends to support the hypothesis.

The results of the questionnaire gave some hints of the role of visualization in problem solving. The descriptions of visualizations seem to fall into two categories, which I will call situational and computational. Situational visualizations may be useful in providing a schema for understanding the problem. Thus an image of a tie rack with a sign saying "1/2 price" might be a help in organizing the relationships among the pieces of information. However, it will not point to a solution. The image of "nine packs of cigarettes in groups of 3's", however, clearly represents one step in the solution process, and would constitute computational visualization.

The availability of two types of images, situational and computational, may help to explain the correlation between subjective experiences of visualization and relative ease of problem solving. Note that situational images can be generated for any problem; even such inherently nonvisual quantities as miles per gallon can produce an image of a car driving down a highway. Such images might not be helpful in reaching a solution, but the subjective experience would be there. If a subject found a problem difficult to grasp, or to solve, the experience would probably be one of searching among a number of different ideas or images, with a sense of confusion. An easier problem would probably result in more stable ideas or images, and these would be more likely to reported by the subject.

Experiment II

Experiment II was devised primarily to replicate Experiment I and to test the hypothesis that more experienced problem solvers would be less influenced by the visualizability of the problems they were working on than less experienced solvers. The rationale for this hypothesis was not that advanced solvers would use visualization more or less frequently than inexperienced problem solvers, but that having a large store of formal strategies to call upon, they would be less dependent on the primitive visualization strategies that the less experienced might use. Thus the prediction was that while the combination of large numbers and continuous quantities would be significantly harder for the less experienced solvers, the more experienced would show no such effect.

Two other factors, not intended to affect visualizability, were built into this experiment. The first was the order in which the information was presented. In one version, the information was presented in the order in which it would be used to solve the problem, while in the other it was scrambled. The second new factor was that some problems used numbers that were chosen to yield integral answers, while others would yield fractional ones. Although these factors produced some intrinsically interesting results (Horwitz, 1980), they did not interact with the visualizability factors and will not be analyzed or discussed in detail here.

Method

Subjects. The subjects came from two populations: 48 subjects from the same college as those in Experiment I and 48 subjects from an elite technical university. The subjects from this university were assumed to be highly competent at the level of the experimental problems simply on the basis of the SAT scores necessary for admission. All subjects were paid at the standard rate.

Materials. The materials consisted of problems based on twelve different story contexts. Each of these stories yielded a problem which was then produced in twelve different forms. Three factors were crossed: size of numbers (large or small), discreteness of quantities (discrete or continuous) and problem form (standard or fractional or scrambled). Three examples of problems from the test follow.

1Aa. A farmer finds that his chickens have eaten 6 bags of feed. An order of feed contains 3 bags. He pays $5 for an order. How much does it cost to feed his chickens?

3Bb. A gardener buys 50 boxes of seeds. The seeds cost $. 30 per box. He uses the seeds to plant 90 rows of vegetables. How much do the seeds for each row cost?

4Cd. A jeweler bought some silver at $90 per ounce. He used the silver to make a 40 inch chain. There were 80 ounces of silver. How much did it cost to make each inch of chain?

Problem 1Aa is an example of small numbers, discrete quantities and standard form; Problem 3Bb uses large numbers, discrete quantities and fractional form; Problem 4Cd uses large numbers, continuous quantities and scrambled form.

Each subject was presented with 12 problems to solve. A fixed order of problem stories was used and a latin square design ensured that each subject saw each of the twelve problems and each of the twelve conditions; across subjects, each problem appeared equally often in each condition.

Again a questionnaire was appended at the end of the test. The main thrust of the questions in this experiment was to collect data on the subjects' perceptions of their own math skills, their problem solving strategies and their understanding of the structure of the problems.

Procedure. The procedure used in this experiment was identical to that of Experiment I, except that subjects were given twelve problems and two practice examples.

Results

The answers to the problems in this experiment were scored differently from those in Experiment I. There, the answers were simply counted right or wrong. Here, simple errors in arithmetic (mainly place value and remainder errors) were counted as correct solutions while errors resulting from incorrect solution plans were counted wrong.

Of the twelve problems each subject solved, only ten were included in the analysis. Two problem stories had to be eliminated, one because of a typographical error, the other because quarters meant to be used as tokens entered into the calculations of many of the College students, giving misleading results. Since the factors were counterbalanced across problems, this loss of data did not interfer with the analysis of the results.

Both subject and item analyses of variance were carried out on error rate and solution time for the two populations separately and combined. Groups of four subjects who, together, represented all quantity conditions made up one megasubject for the subject analyses.

Errors. The most important result of the experiment was the support for the hypothesis that visualizability, as measured by the interaction of large and small numbers with discrete and continuous quantities did affect the two groups differently. The analysis of subject error rates for the two groups showed an interaction of the groups with the small-large and discrete-continuous factors to be significant ($F(1, 22) = 6.09$, $MS = 1.84$, $p < .025$). The University subjects' results showed no interaction between number size and discreteness, while the College results were more like those of Experiment I. The mean error rates are shown in Table 2.

Table 2: Error Rates

	College		University	
	Small	Large	Small	Large
Discrete	28%	26%	4%	6%
Continuous	29%	42%	9%	7%

The overall performance of the two groups shows that difference in competence was as great as expected. The error rate for the College subjects was 31% as compared to 6.5% for the University subjects. (The mean solution times were just as strikingly different with College subjects taking an average of 90.7 seconds to read and solve each problem while the mean for the University subjects was 42.4 seconds.)

As stated previously, the error rates on fractional answers and scrambled order of presentation as analyzed separately are not relevant to the concerns of this paper. It may be worth noting, however, that the lack of main effects or interactions in the University subjects' error rates with respect to the visualizability factors were not due to a ceiling effect. ANOVAs of the University subjects' scores showed a main effect for form of presentation and an interaction between form of presentation and discreteness of quantity.

Questionnaire. The first three questions on the questionnaire dealt with the subjects' perceptions of their levels of skill and confidence. It is clear from the responses that these perceptions closely matched the original assumptions in selecting the subjects as well as the error and solution time data.

The next four questions were designed to discover to what extent each subject population was aware of the structure of the problems and the difficulties entailed in solving them. The overall impression one gets is that the University group had a much better grasp of the structure of the problems and their varying degree of difficulty. They were much more

aware of the factors built into the experiment than the College subjects. The most frequently mentioned variable was order of presentation (University: 48 times, College: 13 times). Altogether the four variables (discrete-continuous, large-small, fractional-integral and standard-scrambled) were mentioned 77 times by University subjects and 26 times by College subjects.

The College subjects were also less able to give accurate and substantive answers to the questions about problem difficulty. Some typical answers were: "The information was easy to understand." "I could relate to them " or "I seemed to be thrown off less by all the extra information." (No problem contained extra information.) University subjects on the other hand gave answers like these: "Information in order, not much transfering of units involved." "It was easy because the numbers were easy to work with." "They could all be solved by unit analysis."

The most striking difference between the two groups was on the last question which asked about solution strategies. Of the 48 University subjects, 30 referred specifically to the technique of unit analysis. Other specific strategies were also mentioned, such as beginning with the question, organizing the data, etc. By contrast, the College students had no common technique. In fact, very few subjects from this group were able to describe very clearly how they had solved the problems. It is also worth noting that twelve of the College students mentioned visualization ("picturing" the problem situation or solution) while only one of the University subjects did so. There was no mention of visualization in the questionnaire.

Finally, it is interesting to note that the two groups differed in their emphasis on solution stages. The College group seemed much more

concerned about the comprehension stage, while the University group tended to focus on the planning stage. More than twice as many responses from the University subjects dealt with the actual solution process (25) than with the comprehension process (13) while for the College subjects these factors were reversed (9 mentioned solution and 31 comprehension).

Discussion

The main hypothesis for this experiment was that subjects with greater experience in mathematics would be less dependent on the visualizability of a problem than those with less experience. The subjects chosen clearly met the criteria of high and low performance on the basis of respective error rates, solution times and self-rating on the questionnaire. The results of Experiment II replicated those of Experiment I as well as supporting the hypothesis. That is, low performance subjects found problems which combined large numbers with continuous quantities much harder to solve than those containing only one or neither of these difficulties, while high performance subjects showed no such effect. These results therefore support the notion that the visualizability of a problem affects its solvability by low performance subjects but not by high performance subjects.

Although these results do not necessarily imply that actual visualization is the mechanism which mathematically naive subjects use to solve arithmetic word problems, there do appear to be some reasons for entertaining this possibility. In particular, the responses to the questionnaire seem to point in this direction. We have not only the actual reports of visualization from the questionnaire in Experiment I, but the data on solution techniques from the questionnaire in Experiment II.

To a certain extent, the solution method used by anyone depends not only on comprehending the particular problem, but on seeing or recognizing it as a problem type (Hinsley, Hayes & Simon, 1977). We see from

the results of the questionnaire that College subjects were much less able to recognize similarities and differences among problems than University subjects. This made it difficult to use formal methods, such as unit analysis, even were they available. It also made it more difficult to learn from early problems how to solve later ones. Greeno (1973) discusses the trade-off between the use of learned techniques which tax long term memory as opposed to "productive thinking" which taxes not only one's insight and ingenuity, but also, if there are no useful heuristics employed, short term memory and processing space. Inexperienced problem solvers, not having access to well learned formal strategies or heuristics, are bound to encounter the latter difficulty. Visualization may well be a technique used by at least some naive problem solvers in an attempt to overcome this difficulty.

Conclusion

In conclusion, I would like to look at some of the educational implications of the more speculative aspects of this research. It may occur to some that if vizualization is of some help to inexperienced problem solvers, then it should be encouraged — maybe even taught. This is, of course a possibility. But a word of caution is necessary. We see that experienced problem solvers, with their well learned strategies, do far better. If we can use visualization as a means to learning these strategies as well as to better comprehend the problem to be solved it would indeed be a worthy enterprise. However, it may well be that dependence on visualization can also act as a hindrance to that learning.

There is a hint in the answers to the first questionnaire, and some evidence in a later experiment not reported here (Horwitz, 1980), that

some students use visualization as an escape from formal problem solving procedures. For example, the subject who reported solving Problem 1a in Experiment I by "seeing" three groups of three packs of cigarettes was using computational visualization as a means of solving the problem without having to invoke the formal operation of division, by simply grouping and counting. Classroom experience as well as experimental results have shown that this is not uncommon. Computational visualization may take two forms: a student may either use it to replace formal operations or to motivate them. That is, the three groups of three can either be counted, or they can be seen as a paradigm for division. Students who remain at the first stage are in serious trouble and may never learn when to apply which arithmetic operation. Students in the second stage will be able to solve arithmetic problems, but may well be handicapped when more abstract levels of thinking are required.

As teachers, then, we must learn to be more aware of the strategies our students are using, and we must be more inventive in helping them move from more primitive to more sophisticated methods of problem solving, not by encouraging or discouraging any particular form of behavior, but by helping the student build on what she or he can already do, to attain a more advanced understanding or more sophisticated technique.

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